ESTIMATION OF THE PARTICLE-ANTIPARTICLE CORRELATION EFFECT FOR PION PRODUCTION IN HEAVY ION COLLISIONS

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Abstract

Estimation of the back-to-back $\pi - \pi$ correlations arising due to evolution of the pionic field in the course of pion production process is given for central heavy nucleus collisions at moderate energies.

1 Introduction

It is usually suggested that in high-energy heavy nucleus collisions an excited volume is formed which undergoes evolution and subsequent decay into free final particles. Particles existing in the excited volume represent a part of the medium being rather quasiparticles than free particles. So the form of their energy spectrum $E_{\mathbf{k}}$ may differ essentially from that of free particles $E^0_{\mathbf{k}} = (\mathbf{k}^2 + m^2)^{1/2}$. It was noted (see [1, 2, 3, 4]) that this feature leads to some modification of the well known identical particle correlations (HBT effect) and also to appearance of specific back-to-back particle-antiparticle correlations (PAC effect). No practical estimations of the pionic PAC effect is known to us.

Below we consider central heavy nocleus collisions at moderate (up to AGS) energies. In this case the excited volume consists mainly from nucleons and pions (at least at the late stage). We consider final state pion correlations. The PAC effect is determined through the evolution parameter $r(\mathbf{k})$ and depends on two factors: first, to what extent the pionic energy $E_{\mathbf{k}}$ is modified and second, what is the characteristic time t_0 of the energy spectrum evolution. Our numerical estimations showed that the pion modification in the course of the hadronic matter evolution (say expansion, cooling) is too slow to give sizable PAC effect. So we consider only the fast breakup of the hadronic matter into free pions (freezeout) as a source of PAC. Usually the breakup is considered as an instantaneous process (neglecting its time duration t_0) thus ensuring maximal PAC effect. However the PAC effect under consideration is sensitive to rather small time intervals of the order of $1fm$. So below we estimate PAC for finite t_0.

2 HBT and PAC effects

In this section we describe in parallel the main features of PAC and HBT correlations taking into account pionic energy modification. To present the results in a simple form we use here a simplified description of the excited hadronic volume (particle source). So the following expressions represent a limiting case of those in Refs. [1, 4] [1]. The volume is suggested to be homogenious, motionless (neglect of the flow), isotopically symmetric and large enough (heavy nuclei). Under these conditions the sigle-particle inclusive cross-section can be written in a simple form:

$$N(\mathbf{k}) = \frac{1}{\sigma}\frac{d\sigma}{d^3k} =< a^\dagger(\mathbf{k})a(\mathbf{k}) >= \frac{V}{(2\pi)^3}\left[n(\mathbf{k}) + (2n(\mathbf{k})+1)\sinh^2 r(\mathbf{k})\right] \qquad (1)$$

where $a^\dagger, a$ are creation and annihilation operators of the final state pions, V is the volume of the source, $n(\mathbf{k})$ is the level occupation number (for example, Bose distribution) and $r(\mathbf{k})$ is the evolution parameter.

[1] The sign of one of two momenta $\mathbf{p}_1, \mathbf{p}_2$ in the right hand side of the second of Eqs.14 in the Ref. [1] must be changed. Then Eqs.13-14 of that paper will be applicable for neutral pions. This erroneous sign appeared because of neglect of the difference between relativistic quantum field and a simple set of quantum oscillators in Ref. [1].

Two-particle inclusive cross-sections are given by

$$\frac{1}{\sigma}\frac{d^2\sigma^{++}}{d^3k_1d^3k_2} = \langle a_1^\dagger a_2^\dagger a_1 a_2\rangle = \langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle + \langle a_1^\dagger a_2\rangle\langle a_2^\dagger a_1\rangle \tag{2}$$

for like sign charged (identical) pions (HBT effect),

$$\frac{1}{\sigma}\frac{d^2\sigma^{+-}}{d^3k_1d^3k_2} = \langle a_1^\dagger b_2^\dagger a_1 b_2\rangle = \langle a_1^\dagger a_1\rangle\langle b_2^\dagger b_2\rangle + \langle a_1^\dagger b_2^\dagger\rangle\langle a_1 b_2\rangle \tag{3}$$

for charged particle-antiparticle ($\pi^+\pi^-$) pairs (PAC) and

$$\frac{1}{\sigma}\frac{d^2\sigma^{00}}{d^3k_1d^3k_2} = \langle a_1^\dagger a_2^\dagger a_1 a_2\rangle = \langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle + \langle a_1^\dagger a_2\rangle\langle a_2^\dagger a_1\rangle + \langle a_1^\dagger a_2^\dagger\rangle\langle a_1 a_2\rangle \tag{4}$$

for neutral pion pairs (both HBT and PAC) with

$$\langle a^\dagger(\mathbf{k}_1)a(\mathbf{k}_2)\rangle = \Big[n(\mathbf{k}) + (2n(\mathbf{k})+1)\sinh^2 r(\mathbf{k})\Big]\, F(\mathbf{k}_1-\mathbf{k}_2) \tag{5}$$

$$\langle a(\mathbf{k}_1)b(\mathbf{k}_2)\rangle = \sinh 2r(\mathbf{k})\left[n(\mathbf{k})+\frac{1}{2}\right] F(\mathbf{k}_1+\mathbf{k}_2) \tag{6}$$

(the same for $< a(\mathbf{k}_1)a(\mathbf{k}_2) >$ in the case of neutral pions). In Eqs.5-6 $r(\mathbf{k})$ is the evolution parameter and the function $F(\mathbf{k}_1 \pm \mathbf{k}_2)$ represents the Fourier transform of the source volume at breakup stage. This is sharply peaked function of $\mathbf{k}_1 \pm \mathbf{k}_2$ (at zero momentum) having characteristic scale of the order of inverse size of the source, this scale being much less than characteristic scales of pion momentum distribution $n(\mathbf{k})$ and evolution parameter $r(\mathbf{k})$. So the last two functions may be evaluated at any of momenta $\mathbf{k}_1, \mathbf{k}_2 \approx \pm\mathbf{k}$ (we suggest that the process is $\mathbf{k} \to -\mathbf{k}$ symmetric, for example collision of identical nuclei at CMS). Evidently pion-pion interaction effects are not present in Eqs.2-4; it is supposed (as usual) that they can be separated from exposed HBT-type correlations.

Relative two-particle correlation functions which are measured in experiment are given by

$$C_2^{++}(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|< a^\dagger(\mathbf{k}_1)a(\mathbf{k}_2) >|^2}{N(\mathbf{k}_1)N(\mathbf{k}_2)} \tag{7}$$

$$C_2^{+-}(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|< a(\mathbf{k}_1)b(\mathbf{k}_2) >|^2}{N(\mathbf{k}_1)N(\mathbf{k}_2)} \tag{8}$$

$$C_2^{00}(\mathbf{k}_1,\mathbf{k}_2) = 1 + \frac{|< a^\dagger(\mathbf{k}_1)a(\mathbf{k}_2) >|^2}{N(\mathbf{k}_1)N(\mathbf{k}_2)} + \frac{|< a(\mathbf{k}_1)a(\mathbf{k}_2) >|^2}{N(\mathbf{k}_1)N(\mathbf{k}_2)} \tag{9}$$

Introducing normalized form-factor of the pre-breakup volume,

$$F(\mathbf{k}_1 \pm \mathbf{k}_2) = \frac{V}{(2\pi)^3} G(\mathbf{k}_1 \pm \mathbf{k}_2), \qquad G(0) = 1 \tag{10}$$

we therefore get

$$C^{++}(\mathbf{k}_1,\mathbf{k}_2) = 1 + G^2(\mathbf{k}_1-\mathbf{k}_2) \tag{11}$$

$$C^{+-}(\mathbf{k}_1,\mathbf{k}_2) = 1 + c(\mathbf{k})G^2(\mathbf{k}_1+\mathbf{k}_2) \tag{12}$$

$$C^{00}(\mathbf{k}_1,\mathbf{k}_2) = 1 + G^2(\mathbf{k}_1-\mathbf{k}_2) + c(\mathbf{k})G^2(\mathbf{k}_1+\mathbf{k}_2) \tag{13}$$

with

$$c(\mathbf{k}) = \left(\frac{\sinh r(\mathbf{k})\cosh r(\mathbf{k})(2n(\mathbf{k})+1)}{\sinh^2 r(\mathbf{k})(2n(\mathbf{k})+1)+n(\mathbf{k})}\right)^2 \tag{14}$$

where Eq.11 gives the usual HBT effect and Eq.12 describes the PAC effect. Both of them contain the same form-factor $G(\mathbf{k})$ (in our approximation) ensuring the same direction $\pi^+\pi^+$ correlations and back-to-back $\pi^+\pi^-$ correlations. Neutral pions show both kinds of the correlations being identical particles and simultaneously antiparticles to themselves.

As it can be seen from Eq.14 the PAC effect is determined through evolution parameter $r(\mathbf{k})$. In turn the evolution parameter depends on time duration t_0 of the pion energy evolution. For very small characteristic times t_0 the expression for $r(\mathbf{k})$ is universal [1],

$$r_m = \frac{1}{2}\ln\left(\frac{E^0_{\mathbf{k}}}{E_{\mathbf{k}}}\right), \qquad t_0 = 0 \tag{15}$$

where $E_{\mathbf{k}}$ is the pion energy at pre-breakup moment and $E^0_{\mathbf{k}}$ is free pion energy. For larger t_0 the evolution parameter lessens and depends on unknown details of the breakup process. For estimation of $r(\mathbf{k})$ we shall use the model expression of Ref. [4]:

$$r(\mathbf{k}) = \frac{1}{2}\ln\left(\frac{\tanh(\pi E^0_{\mathbf{k}}t_0/2)}{\tanh(\pi E_{\mathbf{k}}t_0/2)}\right) \tag{16}$$

Below we estimate the evolution parameter $r(\mathbf{k})$ and the factor $c(\mathbf{k})$ in Eq.14 which determines the strength of the PAC effect.

3 Estimation of PAC in finite nucleon density matter

To evaluate PAC one has to know the pion energy spectrum $E_{\mathbf{k}}$ in finite nucleon density matter. The simplest way to find the energy spectrum is the use of the notion of the pseudopotential [5]. It is determined as an effective potential in which the pions propagate and it is given by the forward scattering amplitude $f(\mathbf{k})$ of the pions on the particles of the medium,

$$U(\mathbf{k}) = -4\pi\rho < f(\mathbf{k}) > \tag{17}$$

where ρ is the density of the medium particles and the amplitude $f(\mathbf{k})$ is averaged over states of the medium particles.

In finite nucleon density matter $f(\mathbf{k})$ is mainly πN scattering amplitude. The main contribution to the amplitude is given here by P-wave scattering dominated by delta resonance. Corresponding momentum dependent effective potential for isotopically

symmetric (number of protons is equal (close) to number of neutrons) matter may be taken in the form (see also Refs. [6, 7]):

$$U(\mathbf{k}) = -\frac{8}{9} f_\Delta^2 \frac{M_\Delta E_\Delta}{Mm^2} \frac{\mathbf{k}^2}{E_\Delta^2 - \mathbf{k}^2 - m^2} \tag{18}$$

with

$$f_\Delta^2/4\pi = 0.37 \tag{19}$$

$$E_\Delta = \left(M_\Delta^2 - M^2 - iM_\Delta\Gamma_\Delta\right)/2M = (2.4 - 0.5i)m \tag{20}$$

where f_Δ is empiric $\pi N\Delta$ coupling constant, m is pion mass, M is nucleon mass, M_Δ and Γ_Δ are mass and width of the delta resonance. Eqs.18-20 represent the sum of the direct and exchange πN scattering diagrams with delta resonance in the intermediate state where we neglected nucleon velocities and omitted terms containing inverse nucleon mass (first order M^{-1}-terms give only a few per cent correction to E_Δ). The pion energy in the nucleon matter is now given by the equation

$$E_\mathbf{k}^2 = m^2 + \mathbf{k}^2 + U(\mathbf{k}) \tag{21}$$

It is shown at Fig.1 for nucleon density ρ equal to one half of the nuclear matter density (energies and momenta are taken in pion mass units).

Let us note that Eq.21 has the form of the pionic dispersion equation with substitution of the effective potential $U(\mathbf{k})$ for pionic polarization operator $\Pi(\mathbf{k}, E_\mathbf{k})$ which depends both on momentum $\mathbf{k}$ and energy $E_\mathbf{k}$. The polarization operator $\Pi(\mathbf{k}, E_\mathbf{k})$ in the same approximation is given by the Eq.18 with substitution of the energy squared $E_\mathbf{k}^2$ for $\mathbf{k}^2 + m^2$ in the denominator of the Eq.18 and the pion spectrum (excitations having pion quantum numbers) is then given by selfconsistent solution of the resulting dispersion equation. At first sight the resulting pionic energy spectrum [6] differs essentially from that of Eq.21, containing at least two branches shown at Fig.1 by dashed lines (original pion and delta-hole mixed states tending to be intersecting ones in the limit of zero coupling constant f_Δ). However, considering effects of the pion energy evolution one must use two pieces of these two branches which correspond to true pion and we return essentially to single branch given by Eq.21, see Fig.1. The vicinity of the would-be intersection point (the resonant point $\mathbf{k}_{res}^2 + m^2 = ReE_\Delta^2, \quad k_{res} = 2.1m$, where these two descriptions still differ, does not contribute in any case (here $r(k_{res}) = 0$ and the imaginary part of the pion energy is maximal and large). All that justifies the use of the pseudopotential $U(\mathbf{k})$ for calculation of the evolution parameter $r(\mathbf{k})$ (introduction of the polarization operator $\Pi(\mathbf{k}, E_\mathbf{k})$ would require a reformulation of the scheme of calculation of the evolution effects).

To evaluate the evolution effects it is necessary to specify the level population $n(\mathbf{k})$, the nucleon density ρ at breakup and the time duration t_0 of the breakup stage of the process. The level population may be approximated by Bose distribution with empiric temperature which we take to be equal to $120MeV$,

$$n(\mathbf{k}) = \left(\exp(E_\mathbf{k}/T) - 1\right)^{-1}, \qquad T = 120MeV \tag{22}$$

It seems reasonable to take the nucleon density ρ to be slightly less than nuclear matter density ρ_n. So below we present estimations for two values of ρ which we consider as limiting ones, $\rho = 0.5\rho_n$ and $\rho = \rho_n$. The time duration t_0 is left as a free parameter.

One more problem is the permissible range of the the pion momenta $\mathbf{k}$ where the potential $U(\mathbf{k})$ given by Eq.18 (that is corresponding scattering amplitude) may be used. Evidently this is a low-energy potential applicable at the most at $k \leq (3-4)m$. Furthermore the imaginary part of the potential must not exceed the difference between quasipion energy and free pion energy. This leaves us small momentum region $k \leq 1.5m$ where the calculation of the PAC effect seems to be reliable. It must be also noted that just above the delta resonant energy ($k_{res} = 2.1m$) there is another source of back-to-back pairs – ρ meson decay ($k = (2.5 \pm 0.5)m$ for free ρ mesons). So the PAC effect under consideration is an additional possible source of the correlated $\pi\pi$ pairs active at lower energies.

Calculation of the evolution parameter $r(\mathbf{k})$ according to Eqs.15-16 together with Eqs.19-21 shows that in the case under consideration it is rather small being zero at $k = 0$ and at $k = k_{res} = 2.1m$ and reaching the maximal values:
$r_{max}(\mathbf{k}) = 0.16$ at $k = 1.6m$ for $\rho = 0.50\rho_n$, $t_0 = 0$;
$r_{max}(\mathbf{k}) = 0.22$ at $k = 1.5m$ for $\rho = 0.75\rho_n$, $t_0 = 0$;
$r_{max}(\mathbf{k}) = 0.30$ at $k = 1.3m$ for $\rho = 1.00\rho_n$, $t_0 = 0$.
The function $r(\mathbf{k})$ decreases fastly when characteristic time t_0 increases and vanishes at $t_0 \approx 2fm$.

The enhancement of single-particle inclusive cross-sections arising due to evolution effect is shown at Fig.2a,b where the distribution $N(\mathbf{k})$, given by Eq.1, over the unenhanced ($r = 0$) value $N_0(\mathbf{k})$ is depicted for different values of the characteristic time t_0 for two nucleon densities $\rho = 0.5\rho_n$ and $\rho = \rho_n$ (pion momenta at Figs.2,3 are taken in pion mass units).

Corresponding results for the factor $c(\mathbf{k})$ in Eqs.12-14, which gives the strength of the PAC effect, are shown at Fig.3a,b.

As can be seen from Fig.3 the PAC effect can be essential if the characteristic breakup time t_0 is small enough ($t_0 < 1fm$). The presence or absence of PAC can serve as a measure of the time duration t_0 about which we have no other information. Contrary to single-particle enhancement, which can have various origin, the PAC effect (if any) can be unambiguously identified through measurement of the excess of say zero rapidity (in CMS) small momentum correlated particle-antiparticle pairs.

4 Conclusions

Estimation of the pionic PAC effect in heavy nucleus collisions shows that it can serve as a substantial source of back-to-back $\pi^+\pi^-$ and $\pi^0\pi^0$ pairs (ensuring also an enhancement of single-particle pion distributions) if breakup (freezout) time is small enough.

References

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Figure captions

Fig.1 Quasipion energy in nucleon medium (with $\rho = 0.5\rho_n$) calculated through pseudopotential (solid line) and polarization operator (dashed lines) together with free pion energy (dotted line).
Fig.2 Enhancement of single-particle inclusive cross-section due to evolution effect for $t_0 = 0,\ 0.5,\ 1.0,\ 1.5 fm$ (from top to bottom). a)$\rho = 0.5\rho_n$, b)$\rho = \rho_n$.
Fig.3 Relative strength of PAC (Eqs.12-14) for $t_0 = 0,\ 0.5,\ 1.0,\ 1.5 fm$ (from top to bottom). a)$\rho = 0.5\rho_n$, b)$\rho = \rho_n$.

Fig1

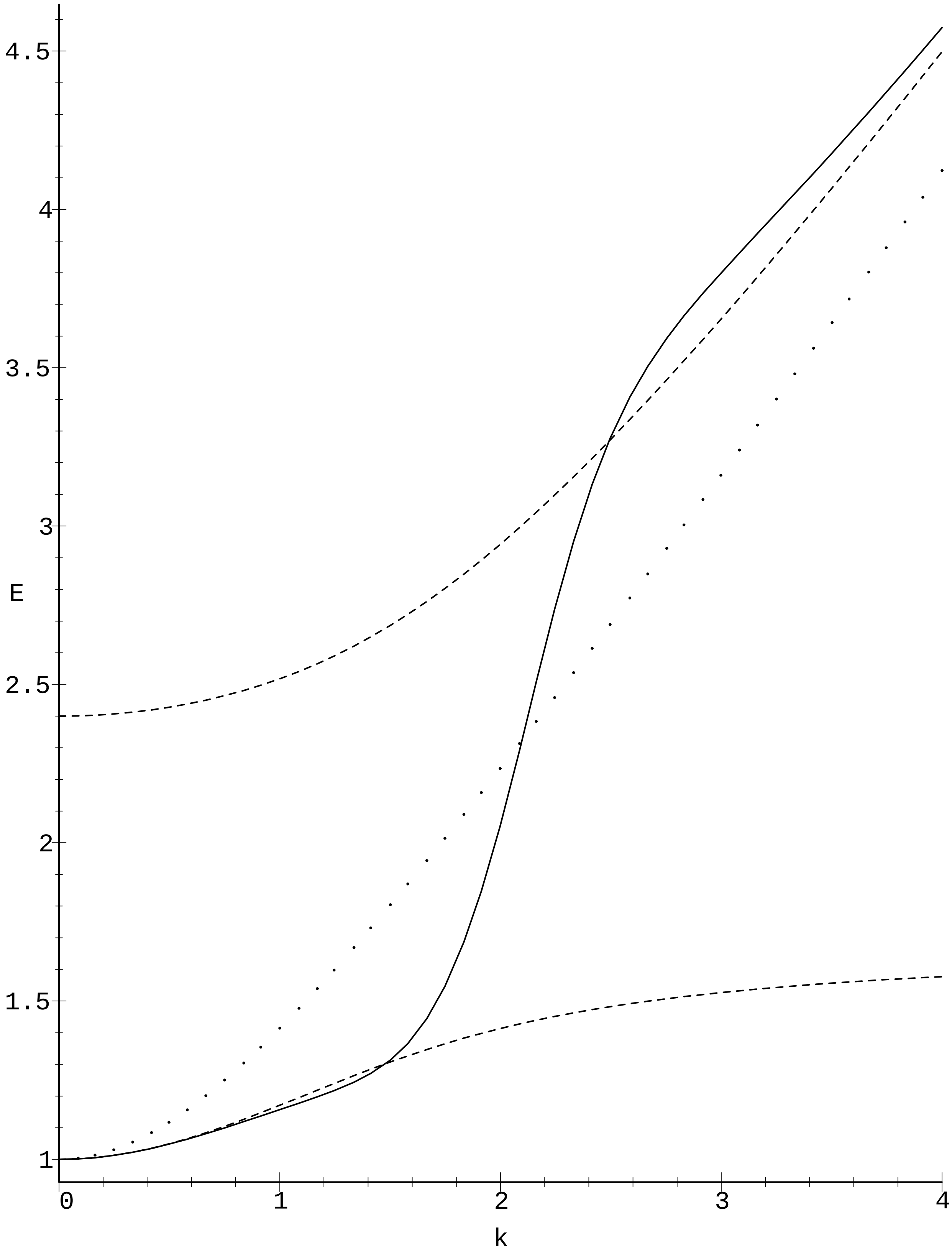

Fig2

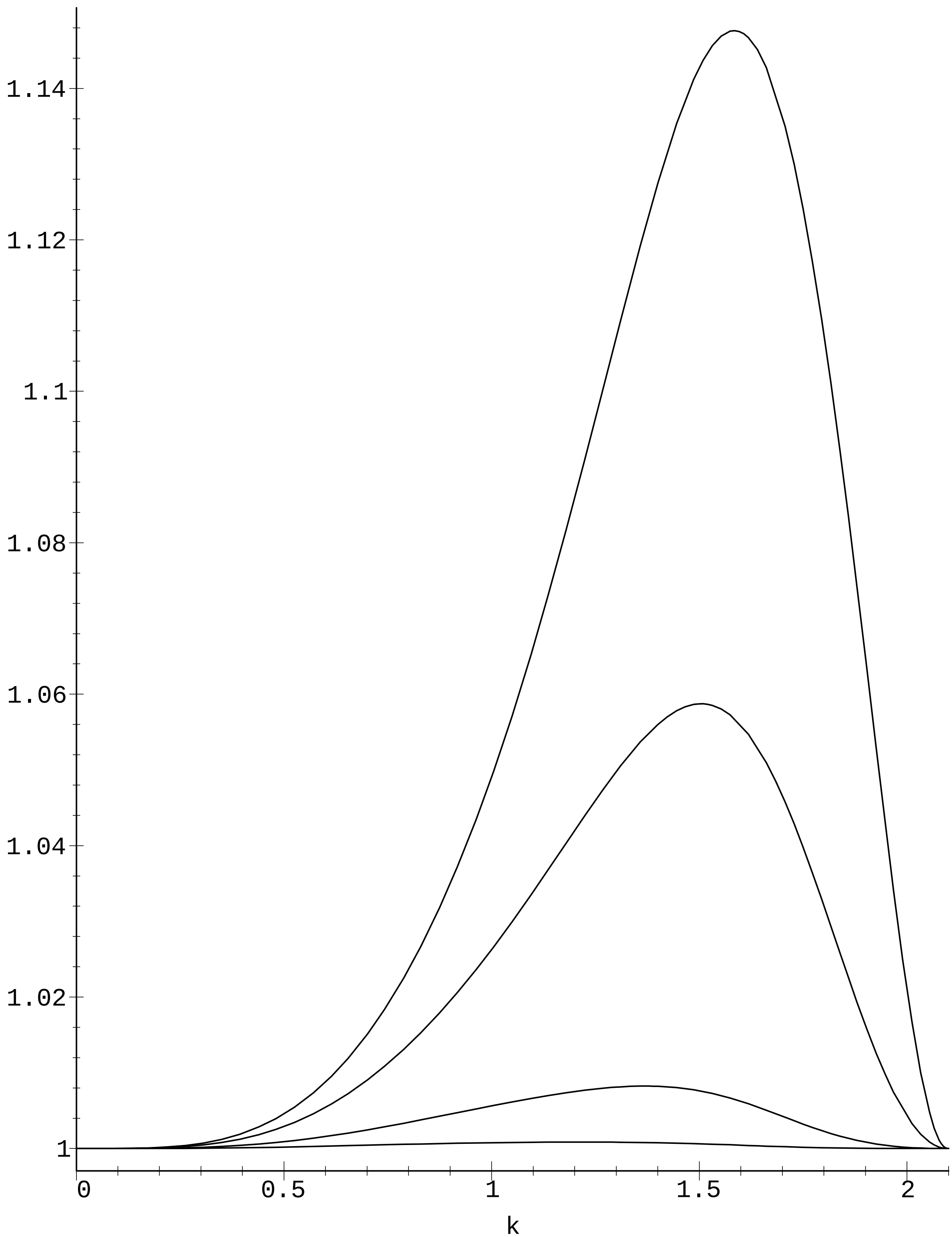

Fig2

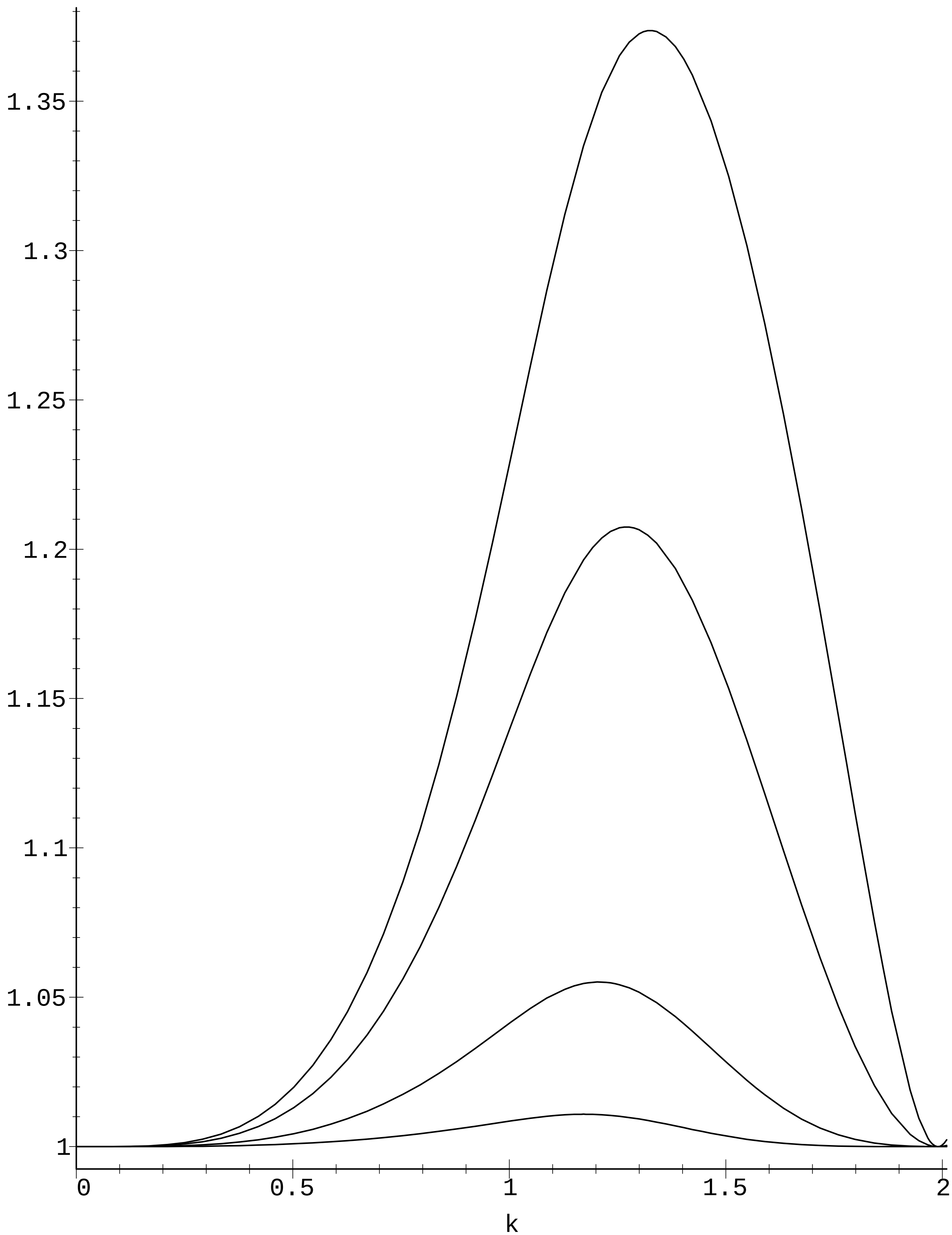

Fig3

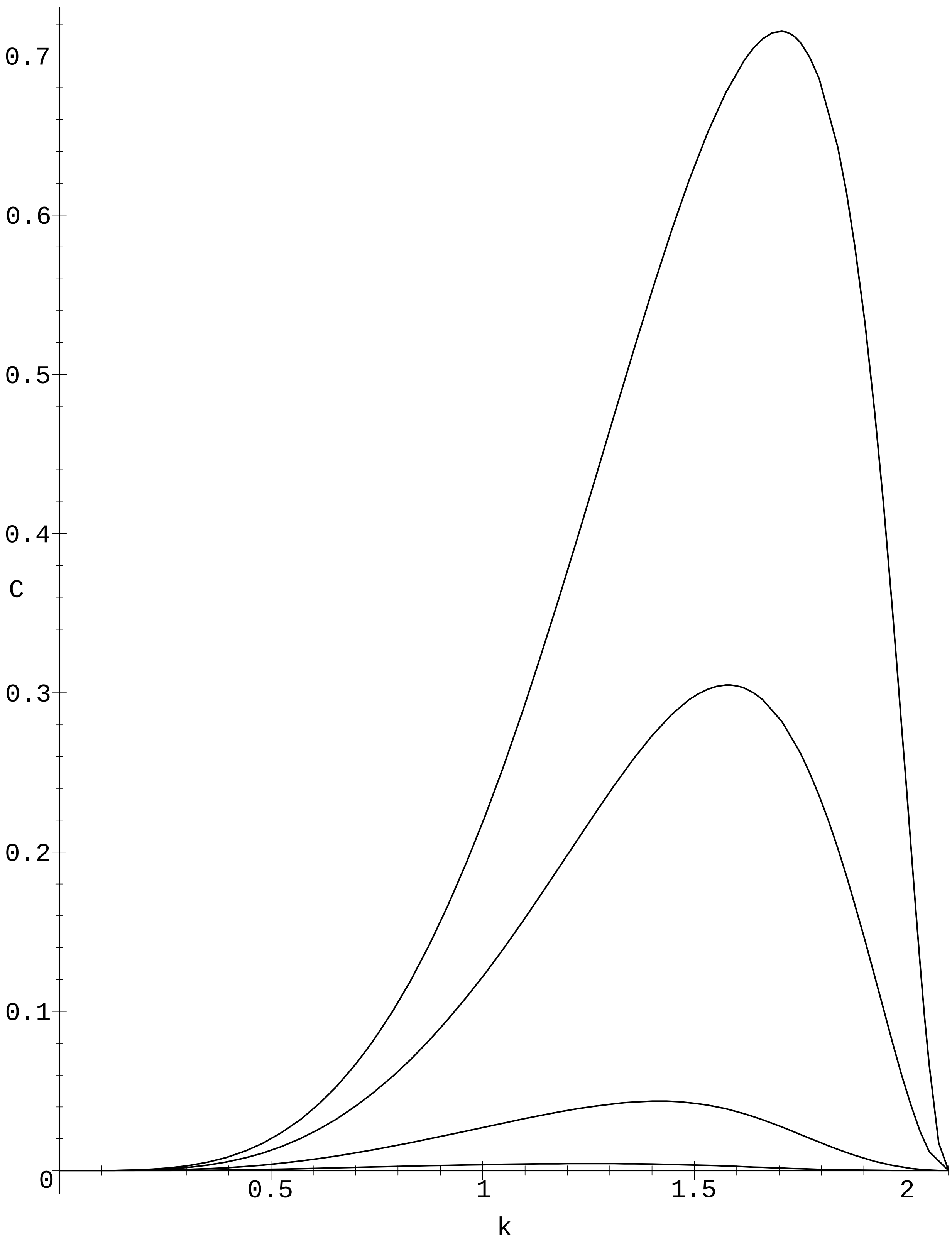

Fig3